

SET 2016
PAPER – III

MATHEMATICAL SCIENCES 040704

Signature of the Invigilator

Question Booklet No.

1. OMR Sheet No.

Subject Code 04

ROLL No.

Time Allowed : 150 Minutes

Max. Marks : 150

No. of pages in this Booklet : 12

No. of Questions : 75

INSTRUCTIONS FOR CANDIDATES

1. Write your Roll No. and the OMR Sheet No. in the spaces provided on top of this page.
2. Fill in the necessary information in the spaces provided on the OMR response sheet.
3. This booklet consists of seventy five (75) compulsory questions each carrying 2 marks.
4. Examine the question booklet carefully and tally the number of pages/questions in the booklet with the information printed above. **Do not accept a damaged or open booklet.** Damaged or faulty booklet may be got replaced within the first 5 minutes. Afterwards, neither the Question Booklet will be replaced nor any extra time given.
5. Each Question has four alternative responses marked (A), (B), (C) and (D) in the OMR sheet. You have to completely darken the circle indicating the most appropriate response against each item as in the illustration.



6. All entries in the OMR response sheet are to be recorded in the original copy only.
7. Use only Blue/Black Ball point pen.
8. Rough Work is to be done on the blank pages provided at the end of this booklet.
9. If you write your Name, Roll Number, Phone Number or put any mark on any part of the OMR Sheet, except in the spaces allotted for the relevant entries, which may disclose your identity, or use abusive language or employ any other unfair means, you will render yourself liable to disqualification.
10. You have to return the Original OMR Sheet to the invigilators at the end of the examination compulsorily and must not carry it with you outside the Examination Hall. **You are, however, allowed to carry the test booklet and the duplicate copy of OMR Sheet** on conclusion of examination.
11. Use of any calculator, mobile phone or log table etc. is strictly prohibited.
12. **There is no negative marking.**

04-16

10. Suppose f_1, f_2 are two monotonically increasing functions on $[a, b]$ and $f = f_1 - f_2$ on $[a, b]$. Then f is :
- (A) Bounded
 (B) Bounded variation on $[a, b]$
 (C) Both (A) and (B)
 (D) Neither (A) nor (B)

11. Suppose A is an onto linear operator on a finite dimensional vector space x then A is :
- (A) One-one (B) Uniformly continuous
 (C) Both (A) and (B) (D) (B) but not (A)

12. Suppose $c(x)$ denote the space of all continuous and bounded real valued functions defined on a metric space x then $c(x)$ is :
- (A) A normed linear space w.r.t. supremum norm
 (B) A metric space w.r.t. the metric induced by supremum norm
 (C) A complete metric space w.r.t. the above metric
 (D) All the above

13. If the characteristic roots of $\begin{vmatrix} a & h & g \\ o & b & o \\ o & a & c \end{vmatrix}$ are a, b, c

then the characteristic roots of $\begin{vmatrix} g & a & h \\ o & o & b \\ c & o & a \end{vmatrix}$ are :

- (A) g, o, a (B) a, b, c
 (C) a, o, c (D) None

14. Suppose v is the vector space of all continuous complex valued functions on $[0, 1]$ with inner product defined by $(f(t), g(t)) = \int_0^1 f(t) \overline{g(t)} dt$. Then Schwarz inequality becomes :

(A) $(f(t) \overline{g(t)})^2 \leq (f(t))^2 (g(t))^2$

(B) $\left| \int_0^1 f(t) \overline{g(t)} dt \right| \leq \int_0^1 (f(t))^2 dt \int_0^1 (g(t))^2 dt$

(C) $\int_0^1 f(t)g(t) dt \leq \int_0^1 f(t) dt \int_0^1 g(t) dt$

(D) $\int_0^1 \overline{f(t)g(t)} dt \leq \int_0^1 \overline{f(t)} dt \int_0^1 \overline{g(t)} dt$

15. Suppose F is a field and V is the set of all polynomials in x of degree 2 or less over F . Let T be a linear transformation on V defined by $T(\alpha_0 + \alpha_1 x + \alpha_2 x^2) = \alpha_1 + 2\alpha_2 x$. Then the matrix of the linear transformation T with respect to the basis $\{1, x, x^2\}$ is :

(A) $\begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 2 & 0 \end{bmatrix}$

(B) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

(D) $\begin{bmatrix} 1 & 0 & 0 \\ 2 & 2 & 0 \\ 3 & 0 & 3 \end{bmatrix}$

16. If $A = \begin{pmatrix} 3 & -1 & 2 \\ -6 & 2 & -4 \\ -3 & 1 & -2 \end{pmatrix}$ then the rank of A^T is :

- (A) 1 (B) 2
(C) 3 (D) 0

17. For what value of λ the system of equations $x+y+z=7$; $x+2y+3z=16$; $x+3y+\lambda z=22$ has a unique solution ?

- (A) 1 (B) 4
(C) 2 (D) 3

18. A characteristic root of $\begin{pmatrix} 0 & -6 & 2 \\ 6 & 1 & -4 \\ -2 & 4 & 2i \end{pmatrix}$ is :

- (A) 0 (B) 1
(C) 2 (D) None

19. The solution of the initial value problem $x' = 4x^{3/4}$, $x(0) = 0$, $t \geq 0$ by using the method of successive approximations is given by :

- (A) $x=1$ (B) $x=t$
(C) $x=t^4$ (D) $x=0$

20. If $\phi(t) = t$ is one of the solutions of the equation $t^2 x'' + tx' - x = 0$ then another linearly independent solution of it is given by :

- (A) $\frac{-1}{2t}$ (B) $\frac{1}{2t}$
(C) $\frac{1}{t}$ (D) $\frac{-1}{t}$

21. If $x(t)$ is a solution of initial value problem $x'' + b_1(t)x' + b_2(t)x = h(t)$; $x(t_0) = x'(t_0) = 0$ and if $x_1(t)$, $x_2(t)$ are linearly independent solutions of homogeneous equation $x'' + b_1(t)x' + b_2(t)x = 0$ and if $w(x_1, x_2)$ is the Wronskian of x_1, x_2 then $x(t)$ is given by :

(A) $\int_{t_0}^t \frac{x_1(s) x_2(t) - x_2(s) x_1(t)}{w(x_1, x_2)(s)} ds$

(B) $\int_{t_0}^t \frac{x_1(s) x_2(t) - x_2(s) x_1(t)}{h(s) w(x_1, x_2)(s)} ds$

(C) $\int_{t_0}^t \frac{(x_1(s) x_2(t) - x_2(s) x_1(t)) h(s)}{w(x_1, x_2)(s)} ds$

(D) $\int_{t_0}^t \frac{x_1(s) x_2(t) - x_2(s) x_1(t)}{h(s)} ds$

22. If the function $f(x, \bar{y}) = (3x + 2y, y_1 - y_2)$ satisfies Lipschitz condition for $|x| < \infty$; $|\bar{y}| < \infty$ then the Lipschitz constant is given by :

- (A) 1 (B) 2
(C) 3 (D) 4

23. The complete integral of PDE $(p^2 + q^2)y = qz$ by Charpit's method is :

- (A) $z^2 - a^2y^2 = (ax + b)^2$
(B) $z - ay = (ax + b)^2$
(C) $z^2 - a^2y = ax + b$
(D) $z - ay = ax + b$

24. A solution matrix of a matrix differential equation $x' = A(t)x$ on I is a fundamental matrix of the system $x' = A(t)x$ on I if:

- (A) $\det \Phi(t) \neq 0, t \in I$
 (B) $\det \Phi(t) = 0, t \in I$
 (C) $\det \Phi(t) = \text{Identity matrix}$
 (D) None of these

25. The rate of convergence of Muller's method is:

- (A) 1.48 (B) 1.84
 (C) 1.88 (D) 1.44

26. For large n the Cholesky method requires:

- (A) $\frac{x^3}{3}$ operations (B) $\frac{x^3}{4}$ operations
 (C) $\frac{x^3}{5}$ operations (D) $\frac{x^3}{6}$ operations

27. The upper bound on the error of interpolation for n tabular points $x_i, i = 1, 2, \dots, n$ and a given point x is

$$|f(x) - p(x)| \leq \max_{a \leq x \leq b} |(x - x_1)(x - x_2) \dots (x - x_n)| M$$

- (A) $M = \max_{a \leq x \leq b} |f^{(n)}(x)| / n!$
 (B) $M = \max_{a \leq x \leq b} |f^{(n-1)}(x)| / (n-1)!$
 (C) $M = \max_{a \leq x \leq b} |f^{(n+1)}(x)| / (n+1)!$
 (D) $M = \max_{a \leq x \leq b} |f^{(1)}(x)| / n!$

28. If $g(x)$ and $g'(x)$ are continuous on an interval I about a root r of the equation $x = g(x)$ then $x_{n+1} = g(x_n), n = 0, 1, 2, \dots$ will converge to the root r if $|g'(x)| < 1$:

- (A) is only sufficient condition
 (B) is only necessary condition
 (C) sufficient and necessary condition
 (D) None of these

29. The Euler equation of the functional

$$J[y(x)] = \int_0^1 (xy + y^2 - 2y^2y') dx \text{ is:}$$

- (A) $xy + y^2 - 2y^2y'$
 (B) $y = \frac{x}{2}$
 (C) $y = \frac{-x}{2}$
 (D) $y = \frac{x^2}{4}$

30. The differential equation of the extremals of the functional $J[y(x)] = \int_0^1 (1 + y'')^2 dx$ is:

- (A) $y' = 0$ (B) $y'' = 0$
 (C) $y''' = 0$ (D) $y^{IV} = 0$

31. The differential equation of the extremals of the functional $J[y(x)] = \int_0^1 (360x^2y - y''^2) dx$ is:

- (A) $y''(x) = 180x$ (B) $y^{IV}(x) = 180x$
 (C) $y^{IV}(x) = 180x^2$ (D) $y^{IV}(x) = 180x^3$

32. The resolvent kernel of the Volterra I.E. with kernel $k(x, t) = 1$ is :
 (A) e^x (B) $e^{\lambda x}$
 (C) $e^{\lambda(x-t)}$ (D) $e^{(x-t)}$
33. The solution of I.E.
 $\phi(x) = 1 + \int_0^x \phi(t) dt$ taking $\phi_0(x) = 0$ is :
 (A) e^{x-t} (B) e^x
 (C) $\sin x$ (D) $\cos x$
34. The resolvent Kernel of the Volterra I.E. with kernel $K(x, t) = e^{x-t}$ is :
 (A) e^{x-t} (B) $e^{(x-t)\lambda}$
 (C) $e^{(x-t)(1+\lambda)}$ (D) $e^{(x-t)\lambda^2}$
35. If $K(x, t)$ is real and symmetric, continuous and identically not equal to zero then all the characteristic constants are :
 (A) Real (B) Imaginary
 (C) Both (A) and (B) (D) None
36. The Lagrange's equation in conservation system is :
 (A) $\frac{d}{dt} \left(\frac{\partial T}{\partial \dot{q}_k} \right) = Q_k + \frac{\partial T}{\partial q_k}$
 (B) $\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{q}_k} \right) = Q_k + \frac{\partial L}{\partial q_k}$
 (C) $\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{q}_k} \right) = \frac{\partial L}{\partial q_k}$
 (D) $\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{q}_k} \right) = \frac{\partial L}{\partial q_k}$
37. The differential equation of damped harmonic oscillation is :
 (A) $\ddot{x} + kx = 0$ (B) $m\ddot{x} - kx = 0$
 (C) $m\ddot{x} + kx = 0$ (D) None
38. The Hamilton's variational principle in mathematical form is :
 (A) $\int_{t_1}^{t_2} L^2 dt = 0$ (B) $\delta \int_a^b L dt = 0$
 (C) $\int_a^b \delta L^2 dt = 0$ (D) $\int_a^b L^3 dt = 0$
39. Let n balls be put into N bags at random then the probability that a particular bag will contain r balls (for $r < n$) is :
 (A) $NC_r \cdot (N-1)^{n-r}/N^n$
 (B) $(N-1)^{n-r}/N^n$
 (C) $NC_r \cdot N^r (N-1)^{n-r}$
 (D) $NC_r (N-1)^{n-r}/N^{n-r}$
40. Which of the following functions can be represented as probability function of a random variable with the given range for x ?
 (A) $f(x) = -\theta e^{-x\theta}, x > 0$
 (B) $f(x) = \frac{15!}{6!8!} x^6 (1-x)^8, 0 < x < 1$
 (C) $f(x) = \frac{8!}{8^4} x^8 e^{-4x}, x > 0$
 (D) $f(x) = \frac{6!8!}{15!} x^6 (1-x)^8, 0 < x < 1$

41. Let $\{x_k : k = 1, 2, \dots\}$ be a sequence of independent random variables with $E(x_k) = 0$ and $V(x_k) = k$. Let

$$S_n = \sum_{k=1}^n x_k, \text{ then as } n \rightarrow \infty :$$

(A) $\frac{S_n}{n^{3/2}} \xrightarrow{P} 0$

(B) $\frac{S_n}{n^{3/2}} \xrightarrow{L} 0$

(C) $\frac{S_n}{n^{3/2}} \cdot \frac{X_n}{n} \xrightarrow{P} 0$

(D) $\frac{S_n}{n^{3/2}} \cdot \frac{X_n}{n} \xrightarrow{L} 0$

42. In a 2^3 factorial design with 3 replicates in 2 blocks each with AB is confounded in replicate I, BC is confounded in replicate II and AC is confounded in replicate III, then the design is :

- (A) Totally confounded
 (B) Partially confounded
 (C) Partially balanced confounded
 (D) Full factorial design

43. The aliases of the effect AB in a 2^{5-2} design with $I_1 = ABE$ and $I_2 = BCD$ are :

- (A) E, ACD, BCDE
 (B) BE, CD, CED
 (C) ABE, BCD, ACED
 (D) None of these

44. A positive recurrent, aperiodic Markov chain is called :

- (A) Pergodic (B) Irreducible
 (C) Absorbing (D) Stationary

45. Match the distribution with property :

- (a) Log normal (i) Mean < Median > Mode
 (b) Pareto (ii) Mean > Median > Mode
 (c) Normal (iii) Mean < Median < Mode
 (d) Weibul (iv) Mean = Median = Mode

(A) (a)-(iii), (b)-(ii), (c)-(iv), (d)-(i)

(B) (a)-(ii), (b)-(i), (c)-(iv), (d)-(iii)

(C) (a)-(i), (b)-(ii), (c)-(iv), (d)-(iii)

(D) None

46. Mean of truncated binomial truncated at $x = 0$ is :

(A) np

(B) np/q^n

(C) $\frac{np}{1 - q^n}$

(D) $\frac{n}{q^n}$

47. The correlation between the i^{th} principal component y_i and the k^{th} variable x_k is :

(A) 0

(B) 1

(C) $\frac{1}{n}$

(D) $C_k \sqrt{\lambda_k} / \sigma_k$

48. The relationship between T^2 and D^2 statistic is (Let $n = n_1 + n_2 - 2$) :

(A) $T^2 = D^2$

(B) $\frac{T^2}{n} = \frac{n_1 n_2 D^2}{n_1 + n_2}$

(C) $n_1 n_2 T^2 = n D^2$

(D) $T^2 = \frac{n_1 + n_2}{n_1 n_2} D^2$

49. Let $x_i \sim B(1, \theta)$, $1 \leq i \leq n$ with Pdf $f(x, \theta) = \theta^x (1 - \theta)^{1-x}$ then Cramer Rao lower bound is:
- (A) θ^2/n (B) $n\theta$
 (C) $n\theta(1 - \theta)$ (D) $\frac{\theta(1 - \theta)}{n}$
50. If x_1, x_2, \dots, x_n be a random sample of size n drawn independently from a population with Pdf $f(x, \theta) = e^{-(x-\theta)}$ then the sufficient statistic for θ is:
- (A) $x_{(1)}$ (B) $x_{(n)}$
 (C) $x_{\binom{n}{2}}$ or $x_{\binom{n+1}{2}}$ (D) $\sum x_i$
51. The Glevenko-Cantelli Lemma is used in the statistic :
- (A) Sign test
 (B) Wilcoxon Sign Rank test
 (C) Run test
 (D) Kolmogorov-Smirnov test
52. If R is the number of runs then $E(R)$ is :
- (A) mn (B) $\frac{mn}{m+n}$
 (C) $1 + \frac{2mn}{m+n}$ (D) $1 + \frac{m+n}{mn}$
53. The maximum likelihood estimator of θ satisfies :
- (A) $L(\theta) = 0$ (B) $\text{Sup}\{L(\theta)\} = 0$
 (C) $\text{Sup}\{L(\theta)\}$ (D) $\text{Inf}\{L(\theta)\} = 0$
54. In a maximization LPP, if a variable corresponding to positive $z_j - c_j$ is entered, the learning variable rule is followed (and the solution space is not unbounded) then:
- (A) The next basic solution will not be basic feasible solution
 (B) The value of the objective function will decrease
 (C) The value of the objective function will increase
 (D) The value of the objective function remains constant
55. The conditional probability of failure is :
- (A) $(R(t) - R(t+L))/R(k)$
 (B) $(R(t) + R(t+L))/R(t)$
 (C) $R(t) - R(t+L)$
 (D) $R(t) + R(t+L)$
56. The dependent variable in logistic regression is called as :
- (A) Bayes variable
 (B) Poisson variable
 (C) Bernoulli variable
 (D) Normal variable
57. A group G has 15 elements. Let $\text{Aut } G$ be its group of automorphisms of G . Then order of $\text{Aut } G$ is :
- (A) 1 (B) 2
 (C) 4 (D) 8

58. Suppose G is a group, $Z(G)$ its centre. The number of groups G such that order of $\frac{G}{Z(G)}$ is 323, is :
- (A) 0 (B) 1
(C) 2 (D) Infinite
59. Let G be a group of order 299. Then the number of subgroups H of G such that index of H in G is 13 is :
- (A) 0 (B) 1
(C) 23 (D) 2
60. The number of normal subgroups of a group G of order 169 is :
- (A) 1 (B) 2
(C) 3 (D) 4
61. Suppose $\langle p(x) \rangle$ is a non-zero prime ideal in $\mathbb{R}[x]$. Then $\frac{\mathbb{R}[x]}{\langle p(x) \rangle}$ is a :
- (A) Field
(B) An integral domain but not a field
(C) Not an integral domain
(D) Not a field
62. An irreducible polynomial in $\mathbb{Q}[x]$ among the following is :
- (A) $3x^3 - 5x^2 + 10x + 15$
(B) $2x^4 + 8$
(C) $x^4 - 4$
(D) $x^6 - 2x^3 - 24$
63. Let $K = \mathbb{Q}(\sqrt{3} + 2i)$. Then the degree of K over \mathbb{Q} is :
- (A) 4 (B) 3
(C) 2 (D) 1
64. Let $f(x)$ be the minimal polynomial of $\sqrt{5} + \sqrt{7}$ over the field \mathbb{Q} . Then the degree of $f(x)$ is :
- (A) 2 (B) 3
(C) 4 (D) 5
65. Consider the topology $\{\emptyset, X, \{x\}, \{x, y\}, \{y, z\}, \{y\}\}$ on $X = \{x, y, z\}$. Then the interior of $\{x, z\}$ is :
- (A) $\{x\}$ (B) $\{z\}$
(C) $\{x, z\}$ (D) X
66. Consider the metric d on \mathbb{R} given by $d(x, y) = \min\{1, |x - y|\}$. Then $\{x : d(x, 0) < 1\}$ is :
- (A) $\{x : |x| < 1\}$ (B) $\{x : |x| \geq 1\}$
(C) $\{x : |x| < \frac{1}{2}\}$ (D) $\{x : |x| \leq \frac{1}{2}\}$
67. An upper bound for the absolute value of $\int_C \frac{e^z}{z+1} dz$, where C is the positively oriented circle $|z| = 5$, is :
- (A) $\frac{e^4}{3}$ (B) $\frac{e^5}{4}$
(C) $\frac{e^5}{5}$ (D) $\frac{e^5}{6}$

68. Suppose $f(z) = u + iv$ is an entire function with $u \leq v$. If $f(i) = 1$ then $2u f(i) - v f(2i) =$
 (A) 2 (B) -2
 (C) -1 (D) 1
69. The radius of convergence of the power series $\sum_{n=1}^{\infty} (-1)^{n-1} n (2z)^{n-1}$ is:
 (A) $\frac{1}{2}$ (B) $\frac{1}{3}$
 (C) $\frac{1}{4}$ (D) $\frac{1}{8}$
70. Suppose $f(z)$ is analytic on $|z| < 1$ with $|f(z)| \leq 1$ and $f(0) = -1$. Then $\int_{-1}^1 f(z) dz =$
 (A) $2i$ (B) $3i$
 (C) $-2i$ (D) $-3i$
71. Let $C : |z| = 1$ be positively oriented. Then $\int_C \operatorname{cosec} z dz =$
 (A) πi (B) $2\pi i$
 (C) $4\pi i$ (D) 0
72. Suppose $f(z) = u + iv$ is an entire function with $v \geq 0$. If $f(2) = 2$ then $\int_{1-i}^{1+i} zf(z) dz =$
 (A) i (B) $2i$
 (C) $3i$ (D) $4i$
73. The image of $x = 1$ under the transformation $w = u + iv = \frac{z}{1-z}$ is:
 (A) $u = 1$ (B) $u = -1$
 (C) $u = 2$ (D) $u = -2$
74. Let C be the circle $|z| = 1$ (positively oriented). Then $\int_C \operatorname{Re}(z) dz =$
 (A) $-2\pi i$ (B) $2\pi i$
 (C) πi (D) $-\pi i$
75. Let C be the circle $|z-1| = 2$ (positively oriented). Then $\int_C \frac{1+z^2+z^4+z^8}{z^9} dz =$
 (A) πi (B) $2\pi i$
 (C) $4\pi i$ (D) $6\pi i$